COVER SHEET

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Mathematical Patterning in Early Childhood Settings

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This paper reports on an investigation of the nature and extent of teacher's knowledge of mathematical patterning and how this knowledge is applied in their daily planning and day-to-day interactions with children. Two case studies indicated that teachers acknowledged the salient role of mathematical patterning and were satisfied with their understanding of patterning processes. However observation of these teachers' classes suggests that there were limited worthwhile patterning opportunities for children.

The pre-school years are widely recognized as a time of profound developmental change in mathematical knowledge and skills (Clements, 2001; Ginsburg & Baron, 1993). Young children's skills in mathematics are developed and made meaningful through processes, such as comparing, counting, symbolizing, classifying, measuring, representing, estimating and patterning. However, despite recommendations for the inclusion of patterning within the curriculum both nationally (e.g., Queensland Studies Authority [QSA], 2004) and internationally (e.g., National Council of Teachers of Mathematics [NCTM], 2000); few research studies have investigated the processes of patterning and its contribution to young children's mathematical development. Likewise, there is scant literature describing the development of children's knowledge of mathematical patterning within the pre-school years, which is the focus of this paper.

The Significance of Mathematical Patterning

Patterning is fundamental to mathematics (Baroody & Coslick, 2000; Steen, 1990). Hence, mathematical patterning is advocated as an essential element of young children's mathematical development and is a central construct of mathematical inquiry (Burns, 2000; Heddens & Speer, 2001; NCTM, 2000; Pengelly, 1992). Most definitions of patterning identify a process of "discovering auditory, visual and motor regularities" (Charlesworth, 2000, p. 190). In particular, patterning ability comprises being able to recognize, extend, create, and copy patterns.

The importance of patterning is advocated in curriculum documents in the pre-school years, such as *The Preschool Curriculum Guidelines* (Queensland School Curriculum Council [QSCC], 1998), and the *Early Years Curriculum* (QSA, 2004). Within the *Preschool Curriculum Guidelines* (QSCC, 1998) patterning is acknowledged in the foundation learning area of 'Thinking'. The guidelines suggest that children "explore patterns in the environment and in language" (p. 42) and that "thinking is used to explore patterns and relationships in the world" (p. 42). In the *Early Years Curriculum* (QSA, 2004), patterning is not restricted to a specific learning area but permeates the curriculum. For example, the document highlights the need for children to engage in experiences that develop patterning abilities, which includes ordering, classifying and sequencing, pattern creation, the recognition of predictable sequences and the associated language.

The emphasis on patterning continues in the compulsory years of schooling, for example, in the *Queensland Mathematics Syllabus Years 1 – 10* (QSA, 2004). In this syllabus, there is a dedicated strand titled 'Patterns and Algebra'. Specific learning outcomes developed for the early years of schooling within this syllabus include the need

for students to "identify, describe and create patterns and change, based on simple rules" (QSA, 2004, p. 23).

Mathematical patterning is also emphasised in international curriculum documents. For example, in New Zealand, patterning is advocated in the Early Childhood Curriculum Te Whariki (Ministry of Education, 1996). Te Whariki incorporates learning statements espousing the need for children to develop concepts, such as, pattern, and the use of pattern strategies for exploring and making sense of the world (Te Whariki, 1996). Thus, this curriculum is comparable to the Queensland early childhood curriculum because pattern knowledge underpins children's thinking and learning processes. In the United States, the *Principles and Standards for School Mathematics* (2000) incorporate a strand of learning dedicated to patterns, functions, and algebra. The patterning outcomes expected for children in Pre-kindergarten to Year Two include the ability to: (a) recognize, describe and extend patterns, and (b) analyse how both repeating and generating patterns are created (NCTM, 2000, p. 90). This is similar to the 'Patterns and Algebra' strand in the Queensland Mathematics Syllabus (QSA, 2004).

Although patterning is integral to the mathematics curriculum in the compulsory years of schooling, it is also a feature of other curricula. Patterning opportunities occur across the curricula via the learning areas of science, art, language, music and physical education. Hence, from children's earliest years, patterning is foundational within and beyond the mathematics curriculum because it assists children to make sense of their everyday world.

Current Research on Mathematical Patterning

Although considerable research exists on many mathematical topics in the early years including number sense (e.g., Bobis, 1996), problem solving (e.g., Cobb et al., 1991), subtraction (e.g., Hughes, 1986), data sense (e.g., Jones, Langrall, Thornton, & Nisbet, 2002), spatial thinking (e.g., Feeney & Stiles, 1996) and inductive reasoning (e.g., English, 2004), there is limited literature on patterning. An exhaustive search revealed that mathematical patterning is generally not studied as a topic area per se, but has featured in the tasks of broader studies, such as a readiness project (Klein & Starkey, 2003) and a reasoning project (English, 2004). An objective of the Berkeley Math Readiness Project (Klein & Starkey, 2003), was to develop a mathematics curriculum for preschool that accounted for socio-economic differences. While pattern duplication tasks were used to assess children's ability to copy linear patterns and pattern extension tasks assessed children's ability to complete linear patterns, children's patterning ability was not studied as an independent phenomenon. Although patterning development was salient to the study, the purpose of the research was to inform early mathematics standards. In another study, English (2004) employed a task called *Attributions and Patterning* in a longitudinal and cross-cultural study of young children's reasoning abilities. English argued that patterning knowledge has an influence on the development of analogical reasoning and that the identification, extension, and generalization of patterns are powerful components of inductive reasoning. While patterning tasks informed English's theory on analogical reasoning and cognitive growth, the development of patterning skills themselves was not studied. Similar to Klein and Starkey (2003), English (2004) employed patterning tasks to assess children's mathematical ability.

Mathematical patterning has also featured in the findings of studies that investigated broader topics. For example, patterning was featured in the results of Ginsburg, Inoue and Seo's (1999) study, which investigated pre-school children's participation in everyday

mathematical activities within a pre-compulsory educative setting. The study examined the types of mathematical competence that underlie young children's everyday activities, such as, what questions they ask, what environments stimulate mathematical learning, and what differences exist between ethnic and racial groups. An aim of the Ginsburg et al. study was to investigate the relative frequency of different types of mathematical activity. In this study, children engaged in pattern and shape activities for 30% of the observed time. This degree of engagement was markedly higher than the other activities identified, such as dynamics, relations, classification and enumeration. Although the phenomenon of patterning was identified in the data, it was not a primary source of investigation.

These studies are examples of how patterning has featured in the literature to date. Patterning processes themselves have not been the focus of investigation. Given the importance of patterning in the early years, a study was designed to develop an understanding of the facets of patterning, and the nature and occurrence of mathematical patterning in the pre-school years.

The Study

This paper reports on one aspect of a multi-site case study (Yin, 2003) that investigated the nature of patterning in the pre-compulsory years of schooling. Case studies are appropriate for producing a descriptive account of phenomena (i.e., patterning) within a real-life context (Yin, 2003). The research reported here focuses on the nature of teachers' knowledge of patterning including evidence of this knowledge within their programming and day-to-day interactions with young children.

Setting

This study was conducted in a preschool and preparatory setting because these sites are typical examples of Queensland children's learning environments in the year prior to the commencement of compulsory schooling. Currently, a full-time preparatory year is being trialled within Queensland to replace the existing preschool program. The two schools chosen for involvement in the study were located in the inner city suburbs of Brisbane. These schools were geographically close and shared similar socio-economic clientele. The preschool was in a state school and operated a five day per fortnight program. The preparatory class was in a private school and conducted a full-time program of five days per week.

Participants

Both the preparatory class and the preschool class consisted of 13 females and 12 males. The students in each of these programs were required to turn five by 31 December of the preceding year to be eligible to attend. The preparatory class teacher, Jo, had 12 years experience in early primary classes. However, this was her first year teaching a full time preparatory class. She had previously taught in preschool centres. Jo identified her professional interests as speech and literacy. Jo also had a full-time teacher aide and an additional special support worker to assist with a boy with mild cerebral palsy. The preschool class was staffed by Linda, a four-year trained early childhood teacher, who was experienced in teaching in preschool. Linda was supported by a full-time teacher aide. She had also previously taught Years 1, 2 and 3, and had been an advisory teacher. In contrast

to Jo, Linda expressed a particular interest in mathematics and had been a member of an early year's mathematics group. Pseudonyms are used in all reporting.

Data collection and analysis

Typical of a case study, multiple sources of data were collected. These data comprised a semi-structured interview with each teacher, copies of their programs and video-taped observation of the classes. The interview questions were designed to elicit information about each teacher's background and their knowledge of patterning. For example, "Do you actively plan for patterning experiences?" Approximately 20 hours of video footage was collected in each class. The video observations represented the typical activities in a pre-compulsory class and included group times, outdoor play, transition games, planned activities, spontaneous play episodes, and snack and lunch times. Teacher's responses to the interview questions were transcribed and analysed for the level of knowledge about patterning and the teaches' employment of this knowledge in their program planning and their interactions with children. The teachers' programs were studied for evidence of explicit planning of patterning experiences. The videos were reviewed to identify any patterning events in typical daily activities. Of particular interest was whether these events were pre-planned or spontaneous, and whether they were child- or teacher-initiated events

Results

Analysis of semi-structured interviews and planning documents suggested that the teachers acknowledged the importance of mathematical patterning and were satisfied with their understanding of the processes involved. Both Jo and Linda identified mathematical patterning as knowledge that should be developed in their students. Although their programs indicated that they had planned some patterning experiences, there was insufficient detail to determine the extent of teachers' knowledge of patterning. For example, one planned experience was simply recorded as "patterning potato prints". Video observations of children's learning opportunities indicated that although the teachers endorsed the importance of patterning, students had few worthwhile patterning experiences. Additionally, teachers failed to capitalise on incidental patterning opportunities. The teachers also exhibited some confusion between patterning and other mathematical concepts, such as line symmetry. The following examples provide 'snapshots' of patterning experiences in the two case study classes.

Patterning in the Preparatory Class

Three patterning experiences were observed in the preparatory classroom. Two of these were planned experiences while one was a child-initiated activity.

The first planned experience required the students to create a pattern on a uniform for a paper doll. To introduce this activity, Jo showed the students various items of clothing to demonstrate patterns. The designs on the clothing were a mixture of shapes, colours, flowers, stripes, checks, hearts and stars. These examples demonstrated random designs and it was very difficult to identify any regularities. It is the repetitive nature of pattern which distinguishes it from random arrangement or design. Jo mentioned the need for repetition when discussing a floral dress, when she observed the "same pattern over and over again in lines" (Figure 1a). However, Jo did not focus on the identification of

repeating elements. She suggested the children could also use “lovely patterns” like stripes, flower patterns, different shapes or checkered patterns on their doll uniforms.

Figure 1a. Floral dress.

Figure 1b. Patterning worksheet.

During the teacher’s introduction, three children discovered patterns on their own clothes. Elizabeth identified an AB pattern around the cuff of her pants. Jo congratulated her and stated “it’s the same over and over again”. Chelsea then showed the class a butterfly on her shirt. Jo helped her locate the matching items on the different sides of the butterfly. Again, Jo congratulated the child and added that she had a lovely pattern on her shirt. Julian then showed the group his shorts, which bore a random design. The random design may have contained a complex pattern however it was not readily identifiable. The examples shown by both the teacher and the children were a combination of patterns (i.e. repeating designs), line symmetry and random designs. However, all were generalised to be patterns. At the end of the activity, only one of the children’s doll uniforms depicted a a repeating design. The child had drawn stripes on the uniform using an ABC pattern and another child had copied this pattern. The other 11 children who participated drew uniforms of random designs with no identifiable repeating elements. The teacher had not provided the students with specific guidelines on how to produce patterns and the examples she provided were not restricted to designs with repeating elements. Thus, children might have been operating from a variety of interpretations of the term “pattern”.

The second planned activity was to complete a patterning worksheet. The worksheet indicated via a colour code, which colour was to be used in which space, and when completed correctly it would create a pattern (Figure 1b). The children were unable to decipher the colour code and their attempts to create patterns largely failed. The children did not identify the repetitious nature of the shapes nor did the teacher suggest any prediction strategies. An identification of regularity makes it possible to predict what lies ahead, however these strategies were not identified or utilised by the teacher. Essential components of linear patterns were neither verbalised to the children nor were examples given.

Whilst Jo planned set patterning tasks, she did not identify other patterning opportunities within incidental events such as collage activities or transition games. However, one child-initiated activity demonstrated an awareness of AB patterning in a painting. Although the teacher praised the child’s painting, she did not acknowledge the pattern within the picture. Thus, although the teacher planned some patterning experiences, there were limited opportunities for children to development their knowledge of patterning.

Patterning in the Preschool Class

In the preschool setting, seven patterning episodes were observed. Two were teacher-planned, three were teacher-initiated, and two were child-initiated.

An example of one of the teacher-planned experiences was a tessellation activity that children completed on a rotational basis. This activity required the children to create tessellating patterns on the carpet using *pattern blocks*. When describing the activity, which had been introduced the previous week, Linda asked the students, "How do you make a tessellating pattern?". The children replied "make it grow". The children were divided into work groups and one group created designs using *pattern blocks*. Four of the six children in the group created designs that grew from a central block (Figure 2a). Another child made a long line of shapes which had some repeating elements in it and another created a random design. While four of the designs grew from a central block, it was unclear whether the children had grasped the concept of tessellation. Tessellations, which are the arrangement of shapes to form spatial patterns, involve patterning skills and knowledge of shape, space and angle. The sophisticated knowledge required of tessellations, such as angle and pattern, is more appropriate for older children. Booker (2004, p.435) suggests that work in tessellations can become "low level busy work and that it can often miss essential underlying mathematical concepts".

Figure 2a. Example of tessellating pattern.

Figure 2b. Child-initiated patterning activity.

The three teacher-initiated experiences required the children to identify and create AB patterns. These consisted of transition activities that had a patterning component, namely numbering off in pairs before moving, sitting in a girl-boy sequence on the carpet, and identifying word patterns in songs. It was evident that Linda provided the class with various opportunities to develop an understanding of the AB form of patterning.

The children demonstrated their knowledge of patterning in a child-initiated activity in which they incorporated AB patterns in borders on Christmas paintings. These paintings were displayed however the teacher did not acknowledge the patterns in them. An additional child-initiated activity occurred when Charlotte created a diamond necklace using a 'hammer and nails' construction game. This activity produced a pattern that Charlotte described as "diamond, funny shape, diamond, funny shape" (Figure 2b). The teacher asked her to assign numbers to go with her pattern and Charlotte pointed to her design and said "one, one, one, one". The teacher then asked, "Any other?" Charlotte replied "red, yellow, red, yellow", which accurately represented the colours of the shapes. The teacher suggested "What about one, two, one, two?". As Linda walked away, Charlotte pointed to her design again and said "one, one, one, one". Thus despite teacher intervention, the child was unable to describe her AB pattern using a numerical representation, although she was able to verbalise the colours of the pattern. The teacher's expectation may be unrealistic for this aged child.

Discussion

Various forms of patterns, from basic repetition through to spatial surface patterns were observed during the case study. While it was evident that teachers planned patterning experiences, it was also evident that both teachers had a limited understanding of the types, levels, and complexity of patterns. Specifically, the teachers showed limited understanding of pattern language, did not seem to be able to integrate pattern development across their curriculum, did not increase the complexity of patterns encountered, and did not use a consistent definition of patterning. The observations also suggested that at times, the teachers inaccurately portrayed patterns in their day-to-day interactions. Thus, the findings of this case study indicates that there is a need to provide increased support to teachers in an effort to enhance teacher's knowledge of patterning. Whilst this study was limited to just two teachers, each of the teachers has more than 10 years teaching experience.

Mathematical patterning provides an essential foundation for many mathematical concepts and processes. As Piaget (1973) attests, children are natural learners and are motivated to learn – their minds are created to learn. Teachers play a myriad of significant roles to assist development. Teachers not only build bridges to assist with the connection between informal and formal mathematical knowledge, they also provide environments and opportunities for children to explore patterning ideas (Clements, Surama, & DiBiase, 2003; NCTM, 2000). There are a variety of strategies and approaches that both Linda and Jo could utilise that would provide opportunities for a more thorough development of children's patterning processes, for example, through teacher modeling, by creating awareness of environmental patterns and by making connections between patterning and incidental events.

Conclusion

The two teachers in the study needed support to improve the patterning opportunities they provided for their students. However, as previously discussed, there is limited research literature on patterning. This is particularly problematic with the moves towards evidence-based curriculum. Evidence-based curriculum refers to an approach, which claims that policy and practice would be able to be validated in terms of convincing evidence about their plausible effects. Slavin suggests that "research in education should ultimately have something to do with improving outcomes for children" (Slavin, 2002, p. 21). Thus, research that provides sound evidence of the power of mathematical patterning is needed to validate the inclusion of patterning in early childhood programs. Additionally, such research would also contribute to the void of knowledge surrounding pattern development and enhance teacher knowledge. Future studies could include research pertaining to the sequential nature of patterning processes, ways to support the development of patterning in young children, and teacher professional development about patterning. These studies would assist in creating 'pattern aware' teachers, who can make informed programming decisions about patterning activities and facilitate worthwhile patterning learning experiences. Additionally, these studies would inform the development of curriculum resource material on patterning.

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